# Lawrence Livermore Laboratory

⊽ REVISITED

R. J. Howerton

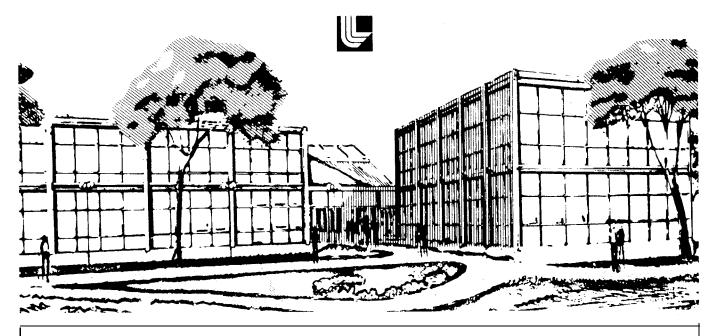
June 25, 1976





This paper was prepared for submission to NUCLEAR SCIENCE AND ENGINEERING

This is a preprint of a paper intended for publication in a journal or proceedings. Since changes may be made before publication, this preprint is made available with the understanding that it will not be cited or reproduced without the permission of the author.



## DISCLAIMER

This document was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor the University of California nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or the University of California, and shall not be used for advertising or product endorsement purposes.

#### ABSTRACT

A method for predicting  $\bar{\nu}(Z, A, E_n)$  is developed and tested against available experimental data ranging from  $\text{Th}^{229}$  to  $\text{Cf}^{249}$ . The only input values required are the charge and mass numbers (Z and A) and the binding energy of the last neutron in the A + 1 nucleus. For incident neutron energies greater than the threshold of multiple chance fission the method is extended by accounting for each fission process separately. This method is an extension of the author's work reported in 1963 and 1971.

# ⊽ REVISITED\*

R. J. Howerton

Lawrence Livermore Laboratory, University of California Livermore, California 94550

June 25, 1976

#### INTRODUCTION

Prediction of the average number of neutrons resulting from neutron-induced  $^{1-5}$  or spontaneous fission has been a recurring question for the past quarter-century. Conventional fission reactors, coupled fusion-fission reactors, and other devices that depend upon the fission process all have in common the figure of merit:  $\bar{\nu}(Z,\,A,\,E_n)$  .  $\sigma_f(Z,\,A,\,E_n)$  [where  $E_n$  is the incident neutron energy]. More recently Meldner et al.  $^6$  addressed the problem for super-heavy elements in the context of prediction of the spontaneous fission  $\bar{\nu}(Z,\,A)$  for very neutron-rich isotopes produced in nuclear explosions. A need for a method of predicting  $\bar{\nu}(Z,\,A,\,E_n)$  for all isotopes with 90  $\leq$  Z  $\leq$  99 was dealt with at a recent International Atomic Energy Agency Advisory Group Meeting  $^7$  in the context of using or disposing of trans-actinides produced in fast-fission breeder reactors.

The early work<sup>1,2</sup> dealt with the observable that  $\bar{\nu}(E_n)$  for each isotope is reasonably represented by a linear function of  $E_n$ , if  $E_n$  is less than the threshold for second-chance fission. The work of Schuster and Howerton<sup>3</sup> developed a truncated pseudo-Taylor series for representation of  $\bar{\nu}(92, A, E_n)$  including a method for extending the incident-neutron energy regime of applicability above the multiple-chance fission thresholds. Howerton's work<sup>4</sup> extended the method of Schuster and Howerton<sup>3</sup> to include a Z dependence by keeping all first order and one cross-product term of the pseudo-Taylor expansion. In both Refs. 3 and 4 the constants for the truncated pseudo-Taylor series were evaluated either directly or indirectly from experimental data. Manero and Konshin<sup>5</sup> presented:

- a) a copious review of experimental data
- b) a simple relationship between the spontaneous fission  $\bar{\nu}(Z, A)$  and  $\bar{\nu}(Z, A-1, Thermal neutron energy)$
- c) a review of the methods of Gordeeva and Smirenkin<sup>8</sup> and Ping-Shin Tu and Prince,<sup>9</sup> both of which works provided methods for estimating  $\bar{\nu}(Z, A, 0)$  as functions of Z and A (Ref. 8 presented a linear function of Z, A and a term related to pairing energy while Ref. 9 used  $Z^2/A^{1/3}$  and  $Z^2/\sqrt{A}$  in power series form)

Throughout this paper E  $_{\rm n}$  0 refers to either 0.0253 eV or Thermal neutron energy.

d) power series fits up to order 5 for  $\bar{\nu}(E_n)$  for various isotopes.

Meldner et al.  $^6$  estimated  $\bar{\nu}_{\rm sp}$  by using the energy balance equation provided by Nix  $^{10}$  which states effectively that the number of neutrons per fission is the quotient of the energy available for neutron emission from the fission fragments divided by the sum of the average separation energy and the average emitted neutron kinetic energy. They then derived average separation energies from mass calculations using Seeger and Howard's formula,  $^{11}$  assumed a value for the average emitted neutron kinetic energy, assumed a Q-value for fission, assumed that the amount of fission energy that goes into photons is equal to the equivalent energy of one neutron per fission. From these assumptions and calculations they calculated  $\bar{\nu}$  from the energy balance equation.

#### THEORETICAL DEVELOPMENT

It is clear that much of the past work has dealt with bits and pieces of predicting  $\bar{\nu}(Z, A, E_n)$  and  $\bar{\nu}_{\rm sp}(Z, A)$ . The development that will be presented here is a continuation of the work of Refs. 3 and 4 but with more detail of the assumptions and reasons for selecting the values about which the pseudo-Taylor expansion is made.

The following assumptions are made:

- 1. The average total kinetic energy of fission fragments is independent of incident neutron energy. This assumption is in agreement with observation. 12
- 2. The average energy per fission that is emitted as prompt photons is constant.
- 3. The total energy released in fission is constant for all isotopes of an element.

With these assumptions, it is further assumed that a Taylor Series expansion in the three variables ( $\mathbf{Z}$ ,  $\mathbf{A}$ ,  $\mathbf{E}_{\mathbf{n}}$ ) is valid. Thus:

$$\bar{\nu}(Z, A, E_n) = T(Z, A, E_n)$$
 (1)

The next problems are to determine the values of  $Z_0$ ,  $A_0$  and  $E_0$  about which the expansion is to be made and the order of the terms in the series that are to be kept. For the  $Z_0$  and  $A_0$  values it seems most reasonable to use an isotope for which many measurements have been reported for  $\bar{\nu}(Z_0, A_0, E_n)$ . The values selected are  $Z_0$  = 92 and  $A_0$  = 235. For  $E_0$  the most reasonable candidate is the fission barrier for the nucleus in question. Since the fission barrier is not an observable per se a better candidate for  $E_0$  is the threshold of the fission reaction ( $E_{Th}$ ), since this quantity can be checked against an observable for those isotopes with a positive energy threshold; e.g.,  $U^{238}$ , and

$$E_{Th} = E_{Barrier} - B_n - 0.9 \text{ MeV}$$
 (2a)

where  $B_n$  is the binding energy of the last neutron in the A + 1 nucleus. The quantity (-0.9 MeV) accounts for the barrier penetration nature of the fission process and was derived from lifetime comparisons between fission and photon emission by Vandenbosch and Seaborg  $^{13}$  who also presented a semi-empirical method for calculating the barrier energy. Comparisons with threshold energies suggest a modification of this value to 0.4. In Ref. 13 the variation of the fission barrier with even-even, even-odd, odd-even and odd-odd fissioning nuclei was dealt with by an argument based on spontaneous fission lifetimes. A more basic approach in light of current knowledge would be to make the argument in terms of pairing energies for the fissioning nucleus. The basic equation of Ref. 13 when combined with Eq. (2a) gives:

$$E_{Th} = 19.0 - 0.36 Z^2/A - B_n - 0.4 MeV$$
 (2b)

for fission of an even-even compound nucleus. It is to be expected that the barrier should be raised due to pairing energy considerations for odd-even and even-odd compound nuclei. From the same argument, one would expect the odd-odd compound nuclei to exhibit twice the effect as the odd-even and even-odd nuclei. Considerations of the binding energy of the last neutron in typical fission fragments lead to an estimate of the pairing energy correction of 0.4 MeV for odd mass compound nuclei. This leads to

$$\bar{\nu}_{Th}(Z, A) = 18.6 - 0.36 Z^2/(A+1)$$

$$+ 0.2[2. - (-1)^{A+1} - (-1)^{Z}] - B_n$$
(2c)

The threshold for fission can be obtained from Eq. (2c) using mass tables 14 derived from experiment or semi-empirical mass formulas such as that of Seeger and Howard. 11 Comparison of the fission thresholds calculated from the formula of Ref. 13 and the measured threshold energies (energy at which the cross section is one-half its plateau value) shows generally good agreement (within a few hundred keV). Of course, the comparison can be made only for those isotopes that have a positive threshold energy.

Using the assumptions that  $\bar{\nu}(Z,A,E_n)$  varies linearly with the excitation of the fission fragments and that the energy released in photons is constant:

$$\bar{v} = kE_{i} = k(E_{T} - E_{K} - E_{\gamma})$$
 (3)

where  $E_i$ ,  $E_T$ ,  $E_K$  and  $E_\gamma$  are the average internal, total, kinetic and photon energies released in fission. If k is the reciprocal of the product of the average separation energy of a neutron from a fission fragment by the average kinetic energy of a fission neutron, Eq. (3) becomes identical in content, but not in form, to the equation of Ref. 10 used in Ref. 6.

Assuming the proportionality of  $E_{\rm K}$  to  ${\rm Z}^2/{\rm A}^{1/3}$  as suggested in Ref. 1 and evaluated more recently in Ref. 12:

$$\frac{dv_{Th}}{v_{Th}} = \frac{dE_T}{E_i} + \frac{E_K}{E_i} \left( \frac{dA}{3A} - \frac{2dZ}{Z} \right) - \frac{dE_{\gamma}}{E_i} \qquad (4)$$

Since  $\rm E_T$  and  $\rm E_\gamma$  are hypothesized to be constant for the isotopes of an element and evaluating Eq.(4) for uranium,  $\rm dE_T$ ,  $\rm dE_\gamma$  and dZ are zero so that for  $\rm U^{235}$ 

$$\frac{d\bar{\nu}_{Th}}{dA} = \frac{E_{K}}{3E_{1}} \frac{\bar{\nu}_{Th}}{A} = \left(\frac{169}{3x22.5}\right) \left(\frac{2.33}{236}\right) \approx 0.02 \quad . \quad (5)$$

Since a pairing energy effect depending on the even odd characteristics of the charge and mass of the compound fissioning nucleus is to be expected, a term which modifies the threshold  $\bar{\nu}$  value is included. Although it is possible to make arguments about the relative magnitudes of the binding energy of the last neutron in even-neutron and odd-neutron fission fragments before emission of fission neutrons and arrive at essentially the same value for the pairing energy contribution, the value used here was obtained from comparing the experimental  $\bar{\nu}(E)$  values for  $U^{235}$  and  $U^{238}$  and is 0.12 neutrons per fission. Thus the  $\nu_{Th}$  value is written:

$$\bar{\nu}_{Th}(Z, A) = 2.33 + 0.06[2. - (-1)^{A+1} - (-1)^{Z}]$$
 (6)

If the average total fission fragment energy equation of Refs. 1 and 12 included a pairing energy term, it should be possible to calculate the constant or, conversely, this implies that a pairing energy term should be included in the equation. The

magnitude of such a term, however, would be such that it would be masked by the uncertainty in the experimental data.

The problem remains to determine the order of truncation of the Taylor Series representation of  $\bar{\nu}(Z,A,E_n)$ . Clearly, the simplest truncation would be to keep only first order terms but, since it is known empirically that for a single element the energy dependence is different for different isotopes, the cross product term involving energy and mass should also be included. Following this simplest approximation (all first order terms plus the energy-mass cross term) the expansion can be written:

$$\bar{v}(Z,A,E_n) = C_0 + C_1(Z-92) + C_2(A-235) + C_3(E-E_{Th}) + C_4(A-235) = (E-E_n) + O(Z,A,E)$$
 (7)

The constant  $C_0$  is evaluated from Eq. (6) for  $U^{235}$ ;  $C_2$  is given by Eq. (5);  $C_3$  is determined from the slope of  $\bar{\nu}(92, 235, E_n)$  experimental values and equals 0.130;  $C_4$  is determined from comparison of fits to experimental data for  $U^{235}$  and  $U^{238}$  and equals 0.006;  $C_1$ , which supplies the Z dependence, is obtained from a comparison of  $Pu^{239}$  and  $U^{235}$  experimental data and equals 0.15. The resulting equation:

$$\bar{\nu}(Z,A,E_n) = 2.39 + 0.06[2. - (-1)^{A+1} - (-1)^{Z}] + 0.15(Z-92)$$
+ 0.02(A-235) + [0.130 + 0.006(A-235)](E - E<sub>Th</sub>)

differs from Eq. (3) of Ref. 4 only in the second term.

The extension of Eq. (2c) and Eq. (8) to applicability in the neutron energy regime where multiple chance fission can occur requires:

- 1. Estimating the probabilities of successive chance fission.
- Estimating the mean kinetic energy of the emitted pre-seisson neutrons.
- 3. Taking proper account of binding energies of the last neutron in the various fissioning isotopes.

After combining the first four terms of Eq.(8) to obtain the value of  $\bar{\nu}(Z, A, E_{Th})$ 

$$\bar{v}_{Th}(Z, A) = 2.39 + 0.06[2. - (-1)^{A+1} - (-1)^{Z}] + 0.15(Z-92) + 0.02(A-235)$$
 (9)

and defining the first factor of the fourth term of Eq. (8) to be

$$\bar{v}_1(A) = 0.130 + 0.006(A-235)$$
, (10)

the following relationship is obtained which applies both above and below the threshold for multiple chance fission.

$$\bar{\nu}(Z,A,E_n) = \sum_{n=0}^{M} R_n \times \{n + \bar{\nu}_{Th}(A - n,Z) + \nu_1(A - n) \\ \times [E_n - E_B(A) + E_B(A - n) - n \times \bar{E}_T(n) \\ - E_{Th}(A - n)]\} .$$
 (11)

where:  $R_n$  are the fractions of the total fission cross section going to each process; i.e.,

$$R_0(E_n) = \frac{\sigma_{\text{direct fission }(E_n)}}{\sigma_{\text{Total fission }(E_n)}}$$
;

$$R_1(E_n) = \frac{\sigma_{nn'f}(E_n)}{\sigma_{Total fission}(E_n)}$$

$$R_2(E_n) = \frac{\sigma_{n,2nf}(E_n)}{\sigma_{\text{Total fission}(E_n)}}$$
; etc.

E<sub>B</sub>(A) = total binding energy of the nucleus with charge
Z and mass A;

 $E_{T}(n)$  = mean energy of pre-scission neutrons;

 $E_{\mathrm{Th}}(A-n)$  = threshold energy of the fission process for the nucleus with charge Z and mass (A-n).

M = the degree of multiple chance fission to be taken
into account.

Estimates of the  $R_n$  and  $E_T(n)$  values were given by Howerton in Ref. 4 and were obtained from considerations of nuclear systematics and available energy.

#### COMPARISON WITH EXPERIMENT

Values of  $\bar{\nu}(Z, A, E_n)$  calculated from Eq.(8) with experimental data agree surprisingly well with measured values, especially since the constants of Eq.(8) are of only one or two

figure significance. Tables 1 through 9 present values of  $\bar{\nu}(Z, A, E_n)$  calculated using Eq. (8) together with appropriate experimental values. The threshold values used were taken from experiment where the fission cross section has a positive threshold and has been measured. For  $\mathrm{U}^{233}$ ,  $\mathrm{U}^{235}$ ,  $\mathrm{Pu}^{239}$  and  $\mathrm{Pu}^{241}$ the threshold was determined such that the zero neutron energy value of  $\bar{\nu}(Z, A)$  agreed with the recommended values of Ref. 5 after renormalization to a value of 3.73 for  $\bar{\nu}$  spontaneous of  $Cf^{252}$ . The experimental values presented in Tables 1 - 9 were likewise renormalized to the same value. Since the threshold is difficult to determine from measured fission cross sections to an accuracy greater than ±100 keV, the observed threshold was adjusted slightly for  $Th^{232}$ ,  $U^{234}$ ,  $U^{236}$ ,  $U^{238}$  and  $Pu^{240}$ such that the weighted mean of the ratio of calculated to experimental  $\bar{\nu}(Z, A, E_n)$  values was essentially unity. The only pathological deviation is noted in the first three values of Table 1 where there is a suggestion of rising  $\bar{\nu}$  with decreasing energy.

To test further the adequacy of Eq. (8) in the order of the energy term, least-squares fitting for several isotopes was undertaken. The experimental data were weighted with the reciprocal of the experimental error and fits were obtained using a standard least-squares method. In no case was a significantly better fit obtained by including higher order (up to order 5) terms in neutron energy.

There have been relatively few measurements of  $\bar{\nu}(Z,A,E_n)$  for isotopes other than those represented in Tables 1 - 9. One

series of zero-energy measurements was reported by Jaffey and Lerner. H9 Table 10 presents their experimental results and calculated values using Eq. (8) and the calculated thresholds from Eq. (2c) for the isotopes not included in Tables 1 - 9. The experimental values of Table 10 were renormalized to the value of 3.73 for the spontaneous fission  $\bar{\nu}$  of Cf<sup>252</sup>. The anomaly associated with the zero energy  $\bar{\nu}$  of U<sup>232</sup> was remarked upon by the experimentalists who stated that there were problems with their apparatus which caused this measurement to be less satisfactory than the other measurements they reported. A single measurement for zero neutron energy  $\bar{\nu}$  has been reported in Ref. 51 for Cf<sup>249</sup>. The experimental value is 4.03 ± 0.04 and the value calculated from Eqs. (2c) and (8) is 4.00.

#### CONCLUSIONS

Expanding the charge, mass, and energy dependence of  $\bar{\nu}$  in the form of a truncated Taylor Series apparently yields a reasonable representation for  $\bar{\nu}(Z, A, E_n)$  if the zero-, first- and one second-order cross term are kept in the truncation. The comparison of calculated with experimental data displayed in Tables 1 - 10 indicate that, in general, agreement can be expected to within 5 percent for isotopes ranging from Th<sup>229</sup> to Cm<sup>245</sup>. The agreement with the zero energy value for Cf<sup>249</sup> to less than one percent may, of course, be fortuitous but the method for predicting  $\bar{\nu}(Z, A, E_n)$  appears to be satisfactory for

a large range in charge and mass. There is no indication that keeping more terms of the Taylor Series representation would yield better agreement with experiment.

Using Eqs. (2c) and (8) for prediction may or may not be valid for very neutron-rich isotopes such as those discussed by Meldner et al.<sup>6</sup> If such predictive calculations are carried out, the results do not agree with those reported in Ref. 6.

### REFERENCES

- \*Work performed under the auspices of the U.S. Energy Research & Development Administration under contract No. W-7405-Eng-48.
- 1. R. B. Leachman, Proceedings of the Second International Conference on the Peaceful Uses of Atomic Energy, held in Geneva (United Nations, New York, 1958) P/2467.
- 2. G. N. Smirenkin et al., Atomnaya Energiya 4, 188 (1958).
- 3. S. H. Schuster and R. J. Howerton, <u>J. Nucl. Energy</u>, <u>Parts</u>
  A/B 18, 125 (1963).
- 4. R. J. Howerton, Nucl. Sci. Eng. 46, 42 (1971).
- 5. F. Manero and V. H. Konshin, Atomic Energy Rev. 10, 637 (1972).
- 6. H. W. Meldner, G. A. Cowan, J. R. Nix and R. W. Stoughton, <u>Phys. Rev. C</u> <u>13</u>, 182 (1976).
- 7. A. Lorenz, "IAEA Advisory Group Meeting on Transactinmin Isotope Nuclear Data," Summary Report INDC(NDS)-74 (1976).
- 8. L. D. Gordeeva and L. D. Smirenkin, Atomnaya Energiya 14, 530 (1963).
- 9. Ping-Shin Tu and A. Prince, <u>J. Nucl. Energy</u> <u>25</u>, 599 (1971).
- 10. J. R. Nix, Phys. Lett. 30B, 1 (1969).
- 11. P. A. Seeger and W. M. Howard, Nucl. Phys. A238, 491 (1975).
- 12. V. E. Viola, Jr., Nucl. Data A 1, 391 (1966).
- 13. R. Vandenbosch and G. T. Seaborg, Phys. Rev. 110, 507 (1958).
- 14. A. H. Wapstra and N. B. Gove, <u>Nucl. Data A</u> 9, 303 (1971).
- 15. D. S. Mather, P. Fieldhouse and A. Moat, <u>Nucl. Phys.</u> <u>66</u>, 149 (1965).
- 16. H. Conde and N. Starfelt, <u>Nucl. Sci. Eng</u>. <u>11</u>, 397 (1961).

- 17. L. I. Prokhorova and G. N. Smirenkin, Yad. Fiz. 7, 961 (1968).
- 18. J. C. Hopkins and B. C. Diven, Nucl. Phys. 48, 433 (1963).
- 19. V. I. Kalashnikov, V. I. Lebedev, L. A. Mikaelyan, P. E. Spivak and V. P. Zakharova, *Proceedings of the Conf. Acad. Sci.*USSR Peaceful Uses Atomic Energy (1955) p. 123.
- 20. V. I. Kalashnikov, V. I. Lebedev, P. E. Spivak and V. P. Zakharova, Proceedings of the Conf. Acad. Sci. USSR Peaceful Uses Atomic Energy (1955) p. 131.
- 21. G. DeSaussure and E. G. Silver, Nucl. Sci. Eng. 5, 49 (1959).
- 22. J. W. Boldeman et al., Nucl. News <u>10</u>, 27 (1967).
- 23. S. C. Fultz, J. T. Caldwell, B. L. Berman, R. L. Bramblett,
  M. A. Kelly, H. D. Wilson, M. S. Coops, R. W. Lougheed,
  J. E. Evans and R. W. Hoff, Phys. Rev. 152, 1046 (1966).
- 24. D. W. Colvin and M. G. Sowerby, Proceedings of the Symposium on Physics and Chemistry of Fission, held in Salzburg (International Atomic Energy Agency, Vienna, 1965) Vol. II, p. 25.
- 25. E. R. Gaerttner, M. E. Jones, D. E. McMillan, J. B. Sampson and T. M. Snyder, Nucl. Sci. Eng. 3, 758 (1958).
- 26. J. E. Sanders, <u>J. Nucl. Energy</u> 2, 247 (1956).
- 27. R. L. Walsh and J. W. Boldeman, <u>J. Nucl. Energy</u> <u>25</u>, 321 (1971).
- 28. D. W. Colvin and M. G. Sowerby, Private Communication (1963).
- 29. P. Fieldhouse, E. R. Culliford, D. S. Mather, D. W. Colvin, R. I. MacDonald and M. G. Sowerby, <u>J. Nucl. Energy</u>, Parts A/B 20, 549 (1966).

- 30. A. DeVolpi and K. G. Porges, Proceedings of a Conference on Nuclear Data Microscopic Cross-Sections and other Data Basic for Reactors, held in Paris (International Atomic Energy Agency, Vienna, 1967) Paper CN-23/40 (1966).
- 31. D. S. Mather, P. Fieldhouse and A. Moat, Phys. Rev. 133, 1403 (1964).
- 32. J. W. Boldeman and R. L. Walsh, J. Nucl. Energy 24, 191 (1970).
- 33. H. Conde, Arkiv Fysik 29, 293 (1965).
- 34. V. G. Nesterov, B. Nurpeisov, L. I. Prokhorova, G. N. Smirenkin and Yu. M. Turchin, Proceedings of the Second International Conference on Nuclear Data for Reactors, held in Helsinki (International Atomic Energy Agency, Vienna, 1970) Vol. II, p. 167.
- 35. J. W. Meadows and J. F. Whalen, "Prompt  $\bar{\nu}_{\rm p}$  of U<sup>235</sup>," Los Alamos Scientific Laboratory, WASH-1068 (1966) p. 21.
- 36. Yu. A. Blyumkina, I. I. Bondarenko, V. F. Kuznetsov, V. G. Nesterov, V. N. Okolovitch, G. N. Smirenkin and L. N. Usachev, Nucl. Phys. 52, 648 (1964).
- 37. M. Soleilhac, J. Frehaut, J. Gauriau, M. Labat, J. Perchereau, J. Nucl. Energy 23, 257 (1969).
- 38. M. V. Savin, Ju. A. Khokhlov and Yu. S. Zamyatnin, Proceedings of the Second International Conference on Nuclear Data for Reactors, held in Helsinki (International Atomic Energy Agency, Vienna, 1970) Vol. II, p. 157.
- 39. H. Conde and M. Holmberg, J. Nucl. Energy <u>25</u>, 331 (1971).

- 40. M. V. Savin, Yu. A. Khokhlov, I. N. Paramonova and V. A. Chirkin, Atomnaya Energiya 32, 408 (1972).
- 41. I. Asplund-Nilsson, H. Conde and N. Starfelt, <u>Nucl. Sci.</u> Eng. 20, 527 (1964).
- 42. D. W. Colvin and M. G. Sowerby, Proceedings of the Second International Conference on the Peaceful Uses of Atomic Energy, held in Geneva (United Nations, New York, 1958)
  Vol. 16, p. 121.
- 43. K. E. Bolodin, V. F. Kuznetsov, V. G. Nesterov, B. Nurpeisov, L. I. Prokhorova, Yu. M. Turchin and G. N. Smirenkin, Atomnaya Energiya 33, 901 (1972).
- 44. D. S. Mather, P. F. Bampton, G. James and P. J. Nind, "Measurements of  $\bar{\nu}_p$  for Pu<sup>239</sup> between 40 keV and 1.2 MeV," Atomic Weapons Research Establishment, Aldermaston report AWRE-0-42/70 (1970).
- 45. R. L. Walsh and J. W. Boldeman, <u>Ann. Nucl. Sci. Eng. 1</u>, 353 (1974).
- 46. H. Conde, J. Hansen and M. Holmberg, <u>J. Nucl. Energy</u> <u>22</u>, 53 (1968).
- 47. M. DeVroey, A. T. G. Ferguson, N. and N. Starfelt, <u>J. Nucl.</u>

  <u>Energy</u>, <u>Parts A/B</u> <u>20</u>, 191 (1966).
- 48. J. Frehaut, M. LeBars and G. Mosinski, report CEA-R-4626 (1974).
- 49. A. H. Jaffey and J. L. Lerner, Nucl. Phys. 145, 1 (1970).
- 50. A. B. Smith, R. K. Sjoblom and J. H. Roberts, <u>Phys. Rev.</u> 123, 2140 (1961).

51. K. E. Volodin, V. G. Nesterov, B. Nurpeisov, G. N. Smirenkin, Yu. M. Turchin, V. N. Kosyakov, L. V. Chistyakov, I. K. Shvetsov, V. M. Shubko, L. N. Mezentsev and V. N. Okolovich, <u>Yad. Fiz.</u> 15, 29 (1972).

Table 1. Comparison of calculated values of  $\bar{\nu}$  for Th<sup>232</sup> with experimental values. The calculated values were obtained using Eq. (8).

NEUTRON ENERGY	NUBAR DNUBAR EXP. EXP.		CALC./ EXP.	YR-REF
1.390 1.420 1.480 1.560 1.610 1.640 1.980 2.230 2.460 2.640 2.860 3.270	2.287 0.076 2.179 0.060 2.146 0.096 2.064 0.073 2.060 0.037 2.100 0.072 2.093 0.055 2.181 0.069 2.154 0.049 2.154 0.049 2.167 0.052 2.246 0.052 2.179 0.054 2.254 0.095 2.383 0.100	0.033 2.088 0.078 2.091 0.045 2.098 0.035 2.107 0.018 2.112 0.034 2.116 0.026 2.134 0.016 2.154 0.033 2.162 0.023 2.182 0.024 2.208 0.025 2.252 0.025 2.252 0.042 2.268 0.031 2.298 0.031 2.335	0.713 0.960 0.975 1.025 1.025 1.009 1.028 1.028 1.028 1.028 1.028 1.028 1.028 1.028	65- 15 65- 16 68- 17 68- 17 65- 16 65- 15 68- 17 65- 16 68- 17 65- 16 68- 17 65- 15 68- 17
3.600 4.020	7.377 F.EE7	0.042 2.333 0.028 2.382	1.002	65- 15

Table 2. Comparison of calculated values of  $\bar{\nu}$  for U<sup>233</sup> with experimental values. The calculated values were obtained using Eq. (8).

			_				
NEUTRON ENERGY		DNUBAR EXP.			CALC./ EXP.	YR-REI	F
	2.498 2.455 2.479 2.456 2.456 2.425 2.467 2.467 2.473 2.473 2.451 2.511	8.031 8.024 8.024 8.024 8.024 8.024 8.024 8.024 8.024 8.024 8.023 8.024 8.023 8.024 8.023 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8 8	0.016 0.0013 0.0	22222222222222222222222222222222222222	1.5999999414092550035799999994140925500136779998999941409255013677999892355785	655557655567677766666776666665553	85901234568787878778588877588585

Table 3. Comparison of calculated values of  $\bar{\nu}$  for  $U^{234}$  with experimental values. The calculated values were obtained using Eq. (8).

NEUTRON ENERGY		DNUBAR EXP.	DNU/ NUBAR	NUBAR CALC.	CALC./ EXP.	YR-RI	ΞF
0.990 1.980 3.000 4.000	2.437 2.641 2.692 2.885	0.033 0.043	0.019 0.012 0.016 0.019	2.606 2.733	1.019 0.987 1.015 0.990	65- 65- 65-	15 15 15 15

Table 4. Comparison of calculated values of  $\bar{\nu}$  for U<sup>235</sup> with experimental values. The calculated values were obtained using Eq.(8).

NEUTRON ENERGY	NUBAR DNUSAR EXP. EXP.		CALC./ EXP.	YR-REF
00000000000000000000000000000000000000	2.396 0.026 2.398 0.020 2.398 0.020 2.398 0.020 2.398 0.020 2.387 0.020 2.387 0.020 2.387 0.021 2.388 0.014 2.389 0.023 2.388 0.014 2.389 0.024 2.389 0.024 2.389 0.014 2.389 0.014 2.389 0.024 2.389 0.024 2.371 0.014 2.389 0.021 2.389 0.041 2.389 0.042 2.449 0.018 2.449 0.018 2.449 0.018 2.440 0.018 2.440 0.018 2.440 0.018 2.440 0.018 2.4420 0.034 2.4420 0.034 2.4420 0.034 2.4420 0.034 2.4420 0.018 2.4420 0.018 2.4420 0.018 2.4421 0.022 2.4423 0.022 2.4429 0.018 2.4429 0.018 2.4429 0.018 2.4429 0.018 2.4438 0.021 2.4438 0.021 2.4438 0.021 2.4438 0.021 2.4438 0.021 2.4438 0.021 2.4438 0.021 2.4438 0.021 2.4438 0.021 2.4438 0.022	0.061 2.3833 0.0661 2.3833 0.0661 2.3833 0.0668 2.3833 0.0668 2.3833 0.0668 2.3833 0.0668 2.3833 0.0668 2.3833 0.0668 2.3833 0.0669 2.3893 0.0669 2.3899 0.0669 2.3899 0.0669 2.3899 0.0669 2.3899 0.0669 2.3899 0.0669 2.3899 0.0669 2.3899 0.0669 2.3899 0.0669 2.3899 0.0669 2.3899 0.0669 2.3899 0.0669 2.3899 0.0669 2.3899 0.0669 2.3899 0.0669 2.3899 0.0669 2.3899 0.0669 2.3899 0.0669 2.3899 0.0669 2.3899 0.0669 0.0	7573341321820100000000000000000000000000000000	246839312334555645656666666666666666666666666666

Table 4 (contd)

			A 150 M		-		-
NEUTRON ENERGY	NUBAR EXP.	DNUBAR EXP.	DNU/ NUBAR	NUBAR CALC.	CALG./ EXP	YR-REF	
88       89       80 <td< td=""><td>2.444444444444444444444444444444444444</td><td>80000000000000000000000000000000000000</td><td>78000000000000000000000000000000000000</td><td>2.468 2.461 2.461 2.465 2.4668 2.4668 2.477 2.477 2.477 2.477 2.477 2.478 2.478</td><td>1.99137962822675333781389841412184516925179371389816288 990990990000000000000000000000000000</td><td>  33   33   33   33   33   33   33   3</td><td>745725572579572874725768877782776876785742587876</td></td<>	2.444444444444444444444444444444444444	80000000000000000000000000000000000000	78000000000000000000000000000000000000	2.468 2.461 2.461 2.465 2.4668 2.4668 2.477 2.477 2.477 2.477 2.477 2.478 2.478	1.99137962822675333781389841412184516925179371389816288 990990990000000000000000000000000000	33   33   33   33   33   33   33   3	745725572579572874725768877782776876785742587876

Table 4 (contd)

NEUTRON ENERGY	NUBAR EXP.		DNU/ NUBAR	NUBAR CALC.	CALC./ EXP.	YR-REF
0.610 0.820 0.820 0.820 0.820 0.820 0.820 0.820 0.920 0.920 0.920 0.920 0.920 0.920 0.920 0.920 0.920 0.920 0.920 0.925	48444334466138858623061173277537795877444419577753624444444445445455554606117327779587744441957775362	00000000000000000000000000000000000000	0.000000000000000000000000000000000000	\text{200}	1.1.0001.099642967555164455947560205554488189908845560200011.0999285504488189908845560200011.099985504488189908845560200011.099999000011.099999000011.099999000011.099999000011.0999999000011.0999999000011.0999999000011.099999999	776576767676767676767676767676767676767

Table 4 (contd)

NEUTRON ENERGY	NUBAR EXP.	DNUBAR EXP.	DNU/ NUBAR	NUBAR CALC.	CALC./ EXP.	YR-REF
11.11.12222222222222222222222222222222	2.5889906286637287735588378553428992099367511748246315666366677668378777777868993675117482663156837877777778689639936751174826631568993675117482263156315689936751174822222222222222222222222222222222222	000000000000000000000000000000000000	80000000000000000000000000000000000000	22222222222222222222222222222222222222	1.00072595512815310807015055552200257736581177 001007259551281531080701500950095000700000000000000000000000	766767676767676767666776766666776788857788577888577666767659999999999
5.980	3.250	0.000	0:014	3.161	0.973	69- 37

Table 5. Comparison of calculated values of  $\bar{\nu}$  for U<sup>236</sup> with experimental values. The calculated values were obtained using Eq. (8).

NEUTRON ENERGY         NUBAR DNUBAR EXP.         DNUZ NUBAR CALC.         CALC.         YR-REF EXP.           0.770         2.433         0.060         0.025         2.388         0.982         71-39           0.820         2.383         0.050         0.021         2.395         1.005         71-39           0.880         2.423         0.050         0.021         2.403         0.992         71-39           0.980         2.453         0.050         0.020         2.417         0.985         71-39           1.080         2.413         0.050         0.020         2.417         0.985         71-39           1.080         2.413         0.050         0.020         2.417         0.985         71-39           1.080         2.413         0.050         0.020         2.417         0.985         71-39           1.500         2.5423         0.040         0.016         2.459         0.590         71-39           1.500         2.532         0.040         0.016         2.542         1.004         71-39           1.500         2.532         0.040         0.016         2.584         1.021         71-39           2.290         2.671         0.05							-		
0.820         2.383         0.050         0.021         2.395         1.005         71-39           0.880         2.423         0.050         0.021         2.403         0.992         71-39           0.980         2.453         0.050         0.020         2.417         0.985         71-39           1.080         2.413         0.050         0.020         2.431         1.007         71-39           1.290         2.483         0.040         0.616         2.459         0.590         71-39           1.500         2.542         0.040         0.616         2.459         0.590         71-39           1.500         2.542         0.040         0.616         2.459         0.590         71-39           1.500         2.542         0.040         0.616         2.459         0.590         71-39           1.690         2.532         0.040         0.616         2.542         1.004         71-39           2.210         2.532         0.040         0.616         2.584         1.021         71-39           2.510         2.572         0.040         0.619         2.595         0.972         71-39           2.590         2.652         <							YR-R	EF	
	0.820 0.880 0.980 1.090 1.290 1.690 1.690 2.290 2.290 2.790 2.790 2.790 4.170	2.383 2.423 2.453 2.413 2.483 2.503 2.532 2.532 2.6571 2.6572 2.701 2.761 2.791 2.830	8.050 8.050 9.050 9.050 9.040 9.040 9.040 9.040 9.040 9.050 9.050 9.050 9.050 9.050	8.021 8.021 9.021 9.021 9.021 9.016 9.016 9.016 9.019 9.019 9.019 9.019 9.019	2.395 2.483 2.417 2.459 2.408 4.514 2.584 2.584 2.584 2.636 2.636 2.799 2.799 2.851	1.005 0.992 0.992 0.990 0.990 0.904 1.004 1.021 9.921 0.994 1.994 0.999 1.007	71- 71- 71- 71- 71- 71- 71- 71- 71- 71-	99999999999999999999999999999999999999	

Table 6. Comparison of calculated values of  $\bar{\nu}$  for U<sup>238</sup> with experimental values. The calculated values were obtained using Eq. (8).

NEUTRON	NUBAR DNUBAI		CALC./	YR-REF
ENERGY	EXP. EXP.	NUBAR CALC.	EXP.	
1.3350 1.3350 1.3350 1.3350 1.3550 1.3550 1.5550 1.	2.486 0.055 2.481 0.055 2.481 0.055 2.526 0.051 2.557 0.049 2.511 0.034 2.573 0.046 2.573 0.046 2.573 0.044 2.573 0.044 2.573 0.044 2.450 0.044 2.450 0.044 2.558 0.041 2.551 0.041 2.551 0.041 2.555 0.041 2.571 0.039	0.017 2.609 0.017 2.701 0.017 2.714 0.018 2.728 0.009 2.736 0.018 2.756 0.018 2.770 0.018 2.770 0.018 2.770 0.019 2.003 0.017 2.013 0.011 2.013	0.000000.00000000000000000000000000000	777766777777667777777667777777667777777

Table 6 (contd)

NEUTRON NUBAR DNUEAR DNU/ ENERGY EXP. EXP. NUBAR		EALC./ EXP.	YR-REF
3.930	2.879 2.890 2.890 2.892 2.901 2.951 2.967 2.967 3.018 3.027 3.027 3.129 3.127	1.013 1.004 0.986 1.019 1.023 1.023 1.025 1.018 1.004 1.004 1.009 1.009 1.009 1.009	59- 37 72- 40 69- 37 72- 40 72- 40 69- 37 72- 40 69- 37 72- 40 69- 37 72- 40 69- 37 72- 40 69- 37 72- 40 69- 37

Table 7. Comparison of calculated values of  $\bar{\nu}$  for Pu<sup>239</sup> with experimental values. The calculated values were obtained using Eq.(8).

NEUTRON ENERGY	NUBAR I EXP.	DHUBAR EXF.	DNU/ NUBAR	NUBAR CALC.	CALC./ EXP.	YR-R	EF
99999999999999999999999999999999999999	22.2.2.2.2.2.2.2.2.2.2.2.2.2.2.2.2.2.2	99000000000000000000000000000000000000	55800993653305595553137538119569504995057956663096569 8000000000000000000000000000000000000	777777777779998889909999999999999999999	8.0001.355447.4267.5590357.603355.2905857.589963.0001.0001.000000011.000335.29005857.589963.0001.0001.000000000000000000000000000	55555665567777777666666677666776666766777777	29V1654V58344344577877737745777378774773744347773

Table 7 (contd)

NEUTRON	NUBAR	DHUBAR	DNU/	NUBAR	CALC./	YR-REF
ENERGY	EXP.	EXP.	NUBAR	CALC.	EXP.	
0.6100 0.6100 0.6500 0.65500 0.65500 0.7700 0.777550 0.85500 0.777550 0.85500 0.99500 0.99500 0.99500 0.99500 0.99500 0.99500 0.007500 0.1.007500 0.1.1000 0.1000 0.1	2.843 2.924 2.924 2.920 3.944 2.933 2.933 2.933 2.940 2.953	000000000000000000000000000000000000	344566956696385695486642663755479877788959856951856 828888888888888888888888888888888888	2.997 3.000	1.000835888712652387408428784057788988217925992899685738871.00080717925992899685738871.0008992899685738878408421792599289968573887840842179259928996857388784084217925992899685738878408421792599289968573887840841.0001.0001.0001.0001.0001.0001.000	70- 69- 69- 69- 70- 69- 70- 69- 70- 69- 70- 69- 70- 69- 770- 70- 70- 70- 70- 70- 70- 70- 70- 7

Table 7 (contd)

NEUTRON	NUBAR DNUBA	NUNG F	NUBAR	CAL <mark>C./</mark>	YR-REF
ENERGY	EXP. EXP.	RABUM	CALC.	EXP.	
1.260500005000000000000000000000000000000	3.055 0.038 2.06	8.815 9.015 9.009 9.0015 9.0015 9.015 9.015 9.015 9.015 9.015 9.016 9.016 9.018 9.018 9.018	33333333333333333333333333333333333333	5.09.09.09.09.09.09.09.09.09.09.09.09.09.	4387835787778388833333333333333333333333

Table 7 (contd)

NEUTRON ENERGY	NUBAR EXP.	DNUBAR EXP.	DNU/ NUBAR	NUBAR CALC.	CALC./ EXP.	YR-R	EF
4.030 4.050 4.220 4.230 4.350 4.430 4.490 4.540 4.700 4.940 5.570	3.539 3.436 3.519 3.512 3.451 3.620 3.511	0.070 0.089 0.089 0.029 0.029 0.022 0.109	0.025 0.025 0.008 0.025	3.471 3.497 3.498 3.517 3.529 3.539 3.546 3.571 3.608 3.626	1.015 0.981 0.994 1.003 0.981 1.023 0.987 1.019		37 38 45 38 37 37 37 37 37
5.910 5.980	3.704 3.686	0.070 0.042		3.757 3.768	1.814 1.822	68- 69-	46 37

Table 8. Comparison of calculated values of  $\bar{\nu}$  for Pu<sup>240</sup> with experimental values. The calculated values were obtained using Eq.(8).

						_
NEUTRON ENERGY	NUBAR DNUI EXP. EXI	BAR DNU/ P. NUBAR	NUBAR CALC.		YR-R	EF
0.100 1.080 1.080 1.150 1.231 1.34640 1.56210 1.56210 1.56210 1.56210 1.56210 1.56210 1.56210 1.56210 1.56240	2.849 0.1 2.859 0.1 3.185	50 0.1390053 0.0553	3.010 02347 00247 00247 00247 0025 0025 0025 0025 0025 0025 0025 002	01.000 91.000	<b>66777777</b> 67777777777777777777777777777	477883388788888888888888888888888888888

Table 9. Comparison of calculated values of  $\bar{\nu}$  for Pu<sup>241</sup> with experimental values. The calculated values were obtained using Eq.(8).

NEUTRON ENERGY	NUBAR DNUB EXP. EXP		NUBAR CALC.	CALC./ EXP.	YR-R	EF
0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0	2.956 8.10 2.849 0.01 2.901 0.12 3.087 0.05 2.859 0.02 3.111 0.11 2.906 0.00 2.860 0.11 3.158 0.03 3.320 0.03 3.330 0.03 3.472 0.04 3.466 0.10 3.629 0.07	5 9.865 4 9.865 4 9.816 5 9.852 6 9.852 6 9.835 8 9.835 8 9.837 9 9.837 9 9.837 9 9.837 9 9.837 9 9.837 9 9.837 9 9.837 9 9.837 9 9.837	2.901 2.901 2.901 2.901 2.901 2.901 2.901 2.901 2.901 3.351 3.351 3.351 3.351 3.351 3.351 3.351 3.351 3.351 3.351	0.981 1.018 1.000 0.940 1.001 0.983 0.983 0.998 1.045 1.023 1.024 1.023 1.028	5965556557644444444444444444444444444444	299610495746886886886886886886886888688888888888
5.880	3.804 0.12	0.032	3.877	1.019	68-	45

Table 10. Comparison of calculations using Eq. (8) with the experimental data of reference 49.

Nuclide	$ar{v}$ Exp	⊽ Calc	ν̄ Calc/ν̄ Exp
Th <sup>229</sup>	2.06 ± .02	1.99	0.966
U <sup>232</sup>	3.10 ± .06	2.37	0.764
U <sup>233</sup>	2.46 ± .01	2.45	0.996
U <sup>235</sup>	2.385 ± .005	2.383	0.999
Pu <sup>238</sup>	2.87 ± .03	2.81	0.979
Pu <sup>239</sup>	2.86 ± .01	2.85	0.997
Pu <sup>241</sup>	2.85 ± .02	2.90	1.018
Am <sup>241</sup>	3.19 ± .04	2.94	0.922
Am <sup>242m</sup>	3.23 ± .02	3.18	0.984
Cm <sup>243</sup>	3.40 ± .05	3.41	1.003
Cm <sup>245</sup>	3.80 ± .03	3.38	0.889